THE ATOMS OF SPACE

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Abstract. In this brief note, it will be shown that space may have hidden properties normally attributed to elementary particles, such as mass and charge. We will also elucidate the thermodynamic properties of these atoms of space by modelling these atoms as ideal gas entities propagating disturbances at the speed of light. We have only demanded consistency in the formulas for circular motion, Einstein's mass-energy eqivalance, wave-particle duality, Planck-Einstein equation, Newton's law of universal gravitation, Schwarzschild solution of general relativity, the Reissner–Nordström metric and black hole thermodynamics. We will then use the adiabatic index formula to elucidate the degrees of freedom of these atoms of space. We will also reinterpret Einstein's theories of relativity, solve the mystery of the double slit experiment, muse on the physical nature of dark energy, and finally uncover a possible blindspot that may have hampered progress in constructing a consistent and complete theory of quantum gravity.

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Date: April 29, 2012.

1. Introduction

Over the past 400 years, since Newton to our present day, we have been trying to understand the nature of our physical reality. We have made some postulates along the way, and have discovered via the formalism of mathematics several successful theories like Newton's theory of universal gravitation, special and general relativity, quantum mechanics and quantum field theory which form the basis of our Standard model of elementary particle interactions. If we ask why they are successful, we will discover that this is due to some underlying symmetry. However, there are major challenges, the most urgent of which is finding a complete and consistent theory of quantum gravity. In our attempt to complete this program, we had to invent structures like strings, branes, loops, extra dimensions, super-symmetry, noncommutative geometery and twistors, to make such a consistent theory realizable. Lately we have also discovered that our universe is made up predominantly of dark matter and dark energy, for which we have no explanation.

Instead of creating epicycles on epicycles of postulates and assumptions, is there another avenue that one can pursue that even a high school student can understand? What has been attempted here is to see how far one can go by looking at the static/invariant solutions of all our successful theories, from geometry of circles to black holes thermodynamics, and see what they are "telling us." Is there a solution to these equations that is complete and consistent, and if so, how can this solution be represented? The aim of this program is not to discover new equations, but to discover a hidden pattern within these equations that may throw some light on the foundational issues of quantum mechanics, special and general relativity, and the contemporary programs of M-/string theory, loop quantum gravity, noncommunative geometry and twistor theory as they aspire to understand the quantum basis or lack thereof of gravity.

Although it has always been suspected from dimensional analysis that the Planck regime is where the fundamental insights lie, no assumption is made a priori in this regard. What will be done is as follows: We will first evoke de Broglie's wave-particle duality, and represent the rest energy of a particle by a photon rotating in a circle. We will then equate the centripetal acceleration of this photon to the gravitational field of the point particle. By so doing, we will discover that this will only admit one unique set of solutions, the traditional Planck quantities. Instead of taking these Planck particles as mathematical constructs, we will discover that if in addition to mass, there is also a charge associated with each Planck particle, then these mathematical objects may represent real physical objects which we can use to model the dynamics of the real world around us. In this light, we will discover that we can model these physical objects as black holes with charge and mass, and we will realize that they are none other than stable extremal Reissner-Nordström black holes.

Then we will proceed to use some to the fundamental relationships of black hole thermodynamics to show that these are actually the fundamental degrees of freedom of space. We will speculate on the nature of the degrees of freedom of these atoms, and provide a representation of these degrees of freedom in terms of spin states, thus providing a physical basis for the Dirac equation and spin networks in loop quantum gravity.

2. The particle-light wave duality

Let m = mass of a point particle at rest, and c = the speed of light. Applying Einstein's mass-energy equivalence equation [1], the rest energy $E_{particle}$ of the point particle is

$$E_{particle} = mc^2 \tag{2.1}$$

Also the energy E_{wave} of a quantum of light in terms of its frequency f and Planck constant h is $E_{wave} = hf$, which can also be expressed in terms of the reduced Planck constant $\hbar$ and angular frequency ω as [2]

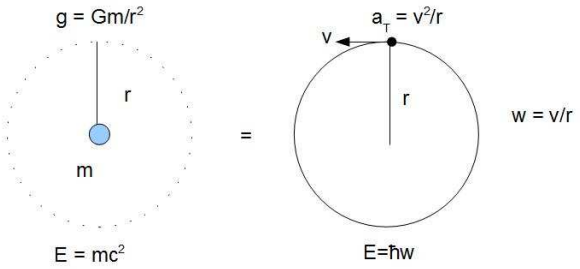
$$E_{wave} = \hbar\omega \tag{2.2}$$

Using the de Broglie's wave-particle duality[3], and letting $E_{particle} = E_{wave}$, and solving for ω, we get

$$\omega = \frac{mc^2}{\hbar} \tag{2.3}$$

Let us now represent the rest energy $E_{particle}$ of the point mass m, with the energy E_{wave} of the quantum of light revolving in a circle of radius r, speed $v = c$, and angular velocity ω as shown in Figure 2.1

Figure 2.1. Particle-wave duality



We will now use the formulas for uniform circular motion [4] to determin the angular velocity ω and centripetal acceleration a_T for this quantum of light. The angular velocity ω is related to the tangential speed v and the radius r of the circle by

$$\omega = \frac{v}{r} \tag{2.4}$$

The centripetal acceleration $a_T$is

$$a_T = \frac{v^2}{r} \tag{2.5}$$

To determine what r is for the circle, let $v = c$ the speed of light, equate (2.3) and (2.4), and solve for r. We get

$$r = \frac{v}{\omega} = \frac{c}{\omega} = \frac{c}{\frac{mc^2}{\hbar}} = \frac{\hbar}{mc} \tag{2.6}$$

which is also the reduced Compton wavelength for the particle.[5]

The centripetal acceleration a_T in terms of the mass of the particle m is determined by substituting (2.6) into (2.5) and equating $v = c$:

$$a_{\mathsf{T}} = \frac{v^2}{r} = \frac{c^2}{r} = \frac{c^2}{\frac{\hbar}{mc}} = \frac{mc^3}{\hbar} \tag{2.7}$$

If, as shown in Figure 2.1, we equate the gravity field strength g at the distance r for the point particle with the centripetal acceleration a_{T} of light, then according to Newton's law of universal gravitation, $a_T = g = \frac{Gm}{r^2} = \frac{mc^3}{\hbar}$. Solving for r,

$$r = \left(\frac{\hbar G}{c^3}\right)^{1/2} \tag{2.8}$$

which is also the Planck length, L_p.[6] By equating (2.6) with (2.8), $r = \frac{\hbar}{mc} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$, we can solve for m:

$$m = \left(\frac{\hbar c}{G}\right)^{1/2} \tag{2.9}$$

which is also the Planck mass, M_p.[6] Finally, for the circle, $\omega = 2\pi f$, where $f =$ frequency or cycles per unit time, $f = \frac{1}{t}$, which is the time for one cycle. Solving for t, and using (2.3) for ω and (2.9) for m, we get

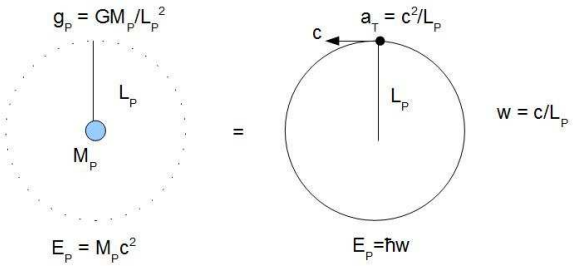
$$t = \frac{(2\pi)}{\omega} = 2\pi(\frac{\hbar}{mc^2}) = 2\pi(\frac{\hbar}{c^2})(\frac{1}{m}) = 2\pi(\frac{\hbar}{c^2})\left(\frac{G}{\hbar c}\right)^{1/2} = 2\pi\left(\frac{\hbar G}{c^5}\right)^{1/2} \tag{2.10}$$

which is also the traditional Planck time[6] x 2π, which we now call t_p. If the momentum of the light in this frame is p, then we can find the angular momentum J of this light wave by using the identity $E = pc$, (2.1) and (2.6). This results in

$$J = pr = (\frac{E}{c})r = (\frac{mc^2}{c})r = (mc)(\frac{\hbar}{mc}) = \hbar \tag{2.11}$$

which implies that the angular momentum of this light representation is $\hbar$.

FIGURE 2.2. Planck particle-wave duality



Thus by evoking 2 equivalence principles, that of 1) rest energy = quantum of energy and 2) the gravitational field = centripetal acceleration of light in the light-frame, we have stumbled upon the zero-point quantum of energy of the gravitational field, $E_p = M_p c^2 = \left(\frac{\hbar c}{G}\right)^{1/2} c^2 = \left(\frac{\hbar c^5}{G}\right)^{1/2}$ (Figure 2.2). At this point, I will postulate that we have also stumbled upon the quantum of space, or the atom of space, whose mass is M_p, whose radius is L_p, an invariant length scale and which also possesses an intrinsic clock whose invariant time period is t_p and whose energy is E_p and angular momentum is J_p. These are summarized below.

$$M_p = \left(\frac{\hbar c}{G}\right)^{1/2}; L_p = \left(\frac{\hbar G}{c^3}\right)^{1/2}; t_p = 2\pi\left(\frac{\hbar G}{c^5}\right)^{1/2}; E_p = \left(\frac{\hbar c^5}{G}\right)^{1/2}; J = \hbar$$

The Schwarzschild radius r_s[7] of the M_p is $r_s = 2\frac{GM_p}{c^2} = 2\ \frac{G}{c^2}\left(\frac{\hbar c}{G}\right)^{1/2} = 2\left(\frac{\hbar G}{c^3}\right)^{1/2} = 2L_p$. Thus the event horizon of the atom of space, where information of the space is stored, is the surface of a sphere of radius $2L_p$. For an atom of space to have an event horizon of sphere of radius L_p and not $2L_p$, I postulate that the atom of space is in fact a gravitational dipole made up of 2 equal masses $m^+$and m^-each of mass $\frac{M_p}{2}$ with the property that they REPEL each other. In order to give them attractive forces to cancel out the repulsive forces, then they both should also have electric charges that are equal and opposite; hence, our atom is also in essence a matter-antimatter gravitational electric dipole. So what should the values of these charges be?

A hint is provided by the Reissner-Nordström metric corresponding to the gravitational field of a charged, non-rotating, spherically symmetric body of mass M, where r_Q is a length-scale corresponding to the electrical charge Q of the mass[8]:

$$r_Q = \left(Q^2\frac{G}{4\pi\epsilon_0 c^4}\right)^{1/2}$$

Since the atom can be considered an extremal black hole when $2r_Q = r_S$, we can solve for Q:

If $2r_Q = r_S = L_p$, then $2\left(Q^2\frac{G}{4\pi\epsilon_0 c^4}\right)^{1/2} = \left(\frac{\hbar G}{c^3}\right)^{1/2}$, and solving for Q, one gets $\frac{1}{2}\left(4\pi\epsilon_0 c\hbar\right)^{1/2}$,which is none other than half the Planck charge .

So if an atom of space is a gravitational and also an electric dipole, we have in one swoop solved both problems mentioned above. Since the strengths of both the gravitation field and electrical field decay with the inverse square distance, then the net force within the atom and also between two atoms has to always be zero, hence there would be no net force causing attraction or repulsion of the atoms of space.

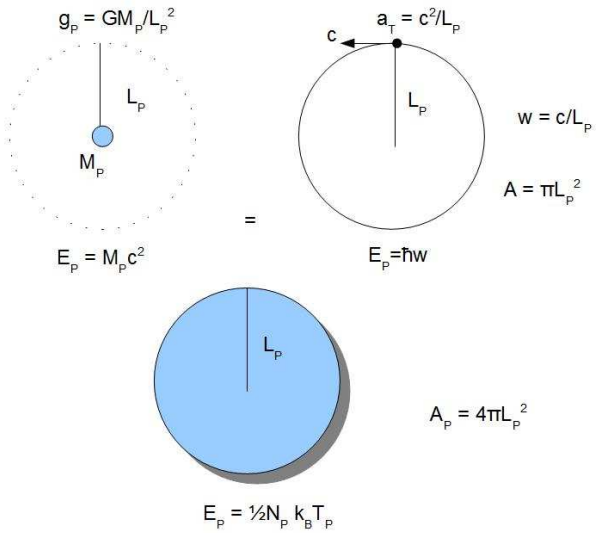
Since the gravitational field is canceled by the electric field for all distances, this may explain why space is thought not to have any mass or charge. However, by postulating an atom of space has an invariable mass-scale, charge-scale, length-scale and time-scale, then space provides the yardstick, by which all of the physical quantities in nature can be measured. This thus begs the question and asks which units are more fundamental, $\hbar$, c, G and ϵ, or M_p, L_p, Q_p, or t_p. Please note that the Planck quantities were derived from the empirical physical constants. If however the Planck quantities are postulated to be more fundamental, then the empirical physical constants are macroscopic consequences of these microscopic physical constants.

3. The particle-black hole duality

We will now use the established results from black hole thermodynamics to determine how much information can be stored in the atom of space and also to elucidate its fundamental degrees of freedom. To do so, we need to find a measure in 2 dimensions that can be used as a gauge or yardstick in 3 dimensions. The holographic principle guides us by relating the scale of the surface of the event horizon $4\pi {L_p}^2$ with the scale of the area corresponding to the particle-light correspondence $A_p = \pi {L_p}^2$. For the event horizon (surface), then the maximum number of bits of information that can be stored is

$$N_p = \frac{A_{surface}}{A_p} = \frac{4\pi {L_p}^2}{\pi {L_p}^2} = 4 \tag{3.1}$$

Figure 3.1. Planck particle-black hole duality



According to the equipartition energy theorem[9], the energy for one degree of freedom is $\frac{1}{2}k_B T$, where k_B is Boltzmann constant and T is the temperature of the surface, so in this case the total energy of the atom E would be

$$E = \frac{1}{2}N_p k_B T = \frac{1}{2}4k_B T = 2k_B T \tag{3.2}$$

Equating this to the energy of the rest mass of the particle (see Figure 3.1 above), we have

$$M_p c^2 = 2k_B T \tag{3.3}$$

Solving forT_p,

$$T_p = M_p \frac{c^2}{2k_B} = \frac{\left[\left(\frac{\hbar c}{G}\right)^{1/2} c^2\right]}{2k_B} = \frac{1}{2}\left(\hbar \frac{c^5}{G{k_B}^2}\right)^{1/2} \tag{3.4}$$

From the particle perspective, the surface gravity,

$$g_p = \frac{GM_p}{{L_p}^2} = \frac{GM_p}{\frac{\hbar G}{c^3}} = \frac{M_p c^3}{\hbar}. \tag{3.5}$$

Inserting (3.3) into (3.5), then $g_p = \frac{2k_B T_p c}{\hbar}$. Thus

$$T_p = \frac{1}{2}\left(\frac{\hbar}{\mathrm{c}k_B}\right) g_p, \tag{3.6}$$

which is none other that Unruh temperature [10] T_{Unruh} x π. (Please note that T_{Unruh}would have been derived if in (3.6) $A_p = {L_p}^2$, instead of $\pi {\mathrm{L}_p}^2$. The latter was chosen so that N= 4 and not 4π.)

At this point, we would like to understand why an atom of space is able to store four bits of information, and to identify the identity of each bit. But before we do so, let us confirm that the atom is actually the smallest unit of entropy.

By using the Bekenstein-Hawking entropy formula for a black hole [11], $S_{\mathrm{BH}} = \frac{1}{4}k_B(\frac{A_{\mathrm{surface}}}{A_p})$, we discover by using (3.1), that

$$S_p = k_B. \tag{3.7}$$

To elucidate the number of degrees of freedom each atom of space represents, we have to now consider a collection of such atoms.

4. The thermodynamics of a collection of atoms of space

Let us now consider a collection of such N atoms of space, confined to a large volume V whose radius $L >> L_p$. We will model this collection as an ideal gas whose density is $\rho = NM_P/V$ and whose pressure $P = Nk_BT/V$, where T is the temperature of this collection of atoms. We will now invoke another principle, the fractal or scale relativity principle, as per Notalle, which equates the temperature T_p of the surface of an atom with the temperature T of the collection of atoms. (Figure 4.1)

To determine the speed v at which a disturbance can propagate through this collection of atoms, we can model this propagation to that of sound in an ideal gas [12]and deduce the adiabatic index γ as follows:

$$v = \left(\gamma \frac{P}{\rho}\right)^{\frac{1}{2}} = \left(\gamma \frac{\left(\frac{Nk_BT_p}{V}\right)}{\left(\frac{NM_p}{V}\right)}\right)^{\frac{1}{2}} = \left(\gamma \frac{k_BT_p}{M_P}\right)^{\frac{1}{2}} \tag{4.1}$$

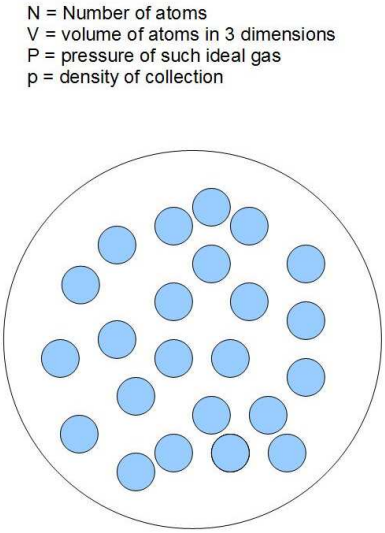
Substituting (3.3) into (4.1), we get,

$$v = \left(\gamma \frac{1}{2}\frac{M_p c^2}{M_P}\right)^{\frac{1}{2}} = c(\frac{\gamma}{2})^{\frac{1}{2}} \tag{4.2}$$

If we equate the speed of propagation of disturbance to the speed of light c in a vacuum, then this implies

$$\gamma = 2.$$

FIGURE 4.1. A collection of atoms of space



Since the adiabatic index is related to the degrees of freedom f_{df} of a gas molecule by the equation [13]

$$\gamma = \frac{(f_{df} + 2)}{2}$$

then

$$f_{df} = 2\gamma - 2 = 2(2) - 2 = 2$$

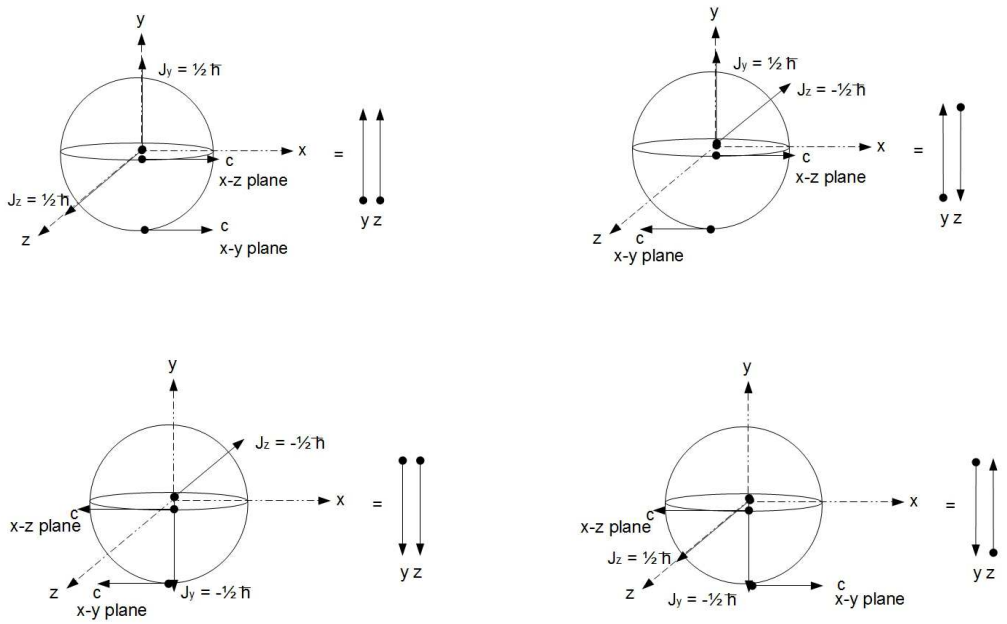
Thus an atom of space has 2 degrees of freedom which can store 4 (q)bits of information!! We can thus model this atom of space, by representing its two matter-antimatter partons with gravitational and electric charges, m^+q^- and m^-q^+, as orthogonal spin $\frac{1}{2}$ states each with J = $\pm\frac{1}{2}$ $\hbar$. This can be modeled as shown in Figure 4.2. Therefore we end up with four degenerate states: $\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\downarrow$, $\downarrow\uparrow$, hence the ability to store 4 (q)bits of information.

To reiterate, since these atoms of space have a unique mass-scale, charge-scale, length-scale and time-scale, these plank particles provides the yardstick, by which all of the physical quantities in nature can be measured. These atoms are equivalent to extremal RN black holes and since they do not produce any Hawking radiation, their stability is guaranteed!![14, 15, 16, 17, 18, 19, 20]

5. EINSTEIN'S THEORIES OF RELATIVITY REVISITED

This model of space is still consistent with the conclusions of special relativity. The postulate that the speed of light is constant in a vacuum finds a natural interpretation, as the speed of information transfer (such as light) from a source to detector is independent of the velocity of the source and that of the detector, as the speed only depends on the physical properties of the atoms of space. I conjecture that it is the flipping of the spin states that is responsible for the traveling

FIGURE 4.2. Model representing the information content of the atom of space



disturbance in space, as space now acts as the ultimate digital storage and transportation medium. Even more so, gravity as an entropic force a la Verlinde [21], or thermodynamic state [22]becomes more credible in its reformulation.

6. The double slit experiment revisited

Let us now model the movement of a particle of mass$m$ with velocity v in a vacuum containing the collection of atoms of space. Since v is always less that c = speed of light, this implies that the speed of disturbance will always be greater than the speed of the particle.

This then throws the double-slit experiment into a new light, as it is only the propagation of the disturbances in the vacuum of the atoms of space that interferes with itself (just like water waves interfering with itself), and thus guides the trailing particle after it passes the slit to its final location on the screen; hence the pilot wave a la Bohm's interpretation of quantum mechanic. Hence quantum mechanics has now acquired its correct interpretation. It is truly remarkable that from a thermodynamics perspective, we have identified the quantum of space, and also solved the mystery of the double-slit experiment at the same time.

7. Triality and the emerging worldview

Thus we see that from the empirical observations of Newton's law of gravitation, Coulomb's law of electrostatics, and the maximum speed of light and the quantum of energy, we have discovered the atoms of space by demanding consistency of

the equations at both the quantum, classical (relativistic) level and also at the statistical (thermodynamics) level. The connection between these levels is coded in the wave-particle principle, and also the holographic principle, respectively. In this sense, our description of space is complete.

This triality between particle framework - wave/light framework - holographic/thermodynamic framework, can be viewed as dealing with 0-dim (0-brane), 1-dim (1-brane = string), and 2-dim objects (2-brane = membrane) points of view, respectively, all equivalent or dual representations or codifications of information of any physical system. String theory uses the light frame perspective and M-theory uses a thermodynamics perspective. I conjecture that Matrix theory uses the 0-brane perspective. Also, quantum loop gravity provides the simple framework (using spin networks) to represent the degrees of freedom of space. If these atoms of space represent the background of space, then we see that both background-dependent and background independent approaches are equivalent, because although the atoms of space have mass and charge, they cancel each other out and appear not to exist. Hence we have something and nothing at the some time. Was this what gauge theory and renormalization theory trying to tell us? In a way, we have discovered the Rosetta stone of physics, the missing link, or as Edward Witten has suggested, the fundamental degrees of freedom for a theory of fundamental interactions or fundamental causation. It would be interesting to see how far this new emerging worldview can go to providing a complete and consistent quantum field theory of gravity or quantum gravity. And finally, we are left asking the question, if these atoms have energy, and are not gravitationally and electromagnetically visible, have we also stumbled on the elementary particles of dark energy?

8. A possible path to non-commutative geometry, twistor theory, unitary theory and E8 theory

A fundamental assumption that may have been a stumbling block to finding a consistent and complete theory of fundamental interactions has to do with the gravitational interactions of matter and antimatter, as there is no experimental evidence to date, given the feebleness of gravitational attraction between elementary particles, to determine conclusively whether it is attractive or repulsive. Several researches think that contrary to the orthodox view that all matter attracts, whether matter or antimatter, they argue based on symmetry and cosmological grounds that if they do repel, then many issues in fundamental physics and cosmology find an elegant solution.[23, 25, 24, 26, 27, 28, 29]

In our model, the atom of space behaves like a gravitational and electromagnetic dipole, with each degree of freedom having mass $m = M_p/2$ and charge $q = Q_p/2$. Since there are 2 degrees of freedom, the energy/mass is divided equally between the two masses m^+ and m^- of opposite charges, matter and antimatter respectively, then the angular momentum for each degree should be $\frac{1}{2}\hbar$. It is the repulsion between the matter-antimatter degrees of freedom that prevents the particle-antiparticle degrees of freedom from annihilating each other. Looking at it from a dual point of view, it is the attraction between the charges that prevent the degrees of freedom from flying apart.

In a recent article, from unitary theory, it appears that the mathematics of gravitons interacting is mathematically identical to two gluons interacting[30]. Could our model of the atom in this note be a physical realization of their mathematical

computations? Also, if these matter-antimatter partons anti-commute, then have we not found a basis for Alain Connes' non-commutative geometry [31] and Sir Roger Penrose's twistor theory[32]? And finally, if we are using geometrical principles and properties of circles of light twisting and turning over each other in well defined ways in 3 dimensional space, can this be extended to higher dimensions and make contact with Garret Lisi's program [33, 34]of constructing a geometric model of elementary particles interactions?

It would be interesting to go back and analyze our super-gravity models with the added feature that antimatter-matter repel, in addition to the feature that matter-matter attract and anti-matter-antimatter attract, and see if those models are now fully re-normalizable. Also, we may be able to make quantum field theory, super-string, loop quantum gravity, E8 theory and eventually quantum gravity consistent and complete.

9. Conclusion

We have arrived at this model of space by only demanding consistency in the formulas for circular motion, Einstein's mass-energy eqivalance, wave-particle duality, Planck-Einstein equation, Newton's law of universal gravitation, Schwarzschild solution of general relativity, the Reissner–Nordström metric and black hole thermodynamics, nothing more or nothing less; hence paving the way for a potentially simple and elegant theory of quantum gravity.

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